Vector Calculus 20E, Spring 2012, Lecture B, Midterm 2

Fifty minutes, four problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let γ be the piece of the curve $y^2 = x^3$ which goes from $(0,0)$ to $(1,1)$. Calculate

$$\int_\gamma x^2 y dx - xy dy.$$

2. Let γ be the boundary of the standard square $[-1,1] \times [-1,1]$, oriented anticlockwise. Use Green's theorem to calculate

$$\int_\gamma (x - y^2)dx + (x^3 + y^4)dy.$$

3. Let Σ be the standard unit sphere, oriented using the outward normal, and let $\mathbf{F}$ be the vector field $\mathbf{F}(x,y,z) = (-x, y, z)$. Calculate (without using Gauss' theorem) the surface integral

$$\int_\Sigma \mathbf{F}.\mathbf{dS}$$

4. Let γ be the circle $x^2 + y^2 = 5$, oriented anticlockwise, and lying in the plane $z = 3$ inside $\mathbb{R}^3$. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x,y,z) = (x^2 \cos x, y^2 \cos y, xyz)$. Use Stokes' theorem to calculate

$$\int_\gamma \mathbf{F}.\mathbf{dS}$$

Vector Calculus 20E, Spring 2013, Lecture A, Midterm 2

Fifty minutes, four problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let γ be the boundary of the square with vertices $(0,0),(1,0),(0,1),(1,1)$, oriented anticlockwise. Calculate

$$\int_\gamma (x^2 - y^2)dx + (x^2 + y^2)dy.$$

2. Let Σ be the unit upper hemisphere $x^2 + y^2 + z^2 = 1, z \geq 0$. Calculate

$$\int_\Sigma z^4 \,\mathrm{dA}.$$

3. Let Σ be the surface given by $x^2 + y^2 = 4$ and $-1 \leq z \leq 1$, oriented using the outward normal, and let $\mathbf{F}$ be the vector field $\mathbf{F}(x,y,z) = (x,y,z)$. Calculate

$$\int_\Sigma \mathbf{F}.\mathbf{dA}$$

4. Let Σ be the surface given by $y = 9 - x^2 - z^2$ and $y \geq 0$, with normal vector pointing in the direction of increasing y. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x,y,z) = (2xyz + 5z, \cos(yz), x^2y)$. Calculate

$$\int_\Sigma (\nabla \times \mathbf{F}).\mathbf{dA}$$